Wrinkling of a stretched thin sheet

Eric Puntel[*], Luca Deseri[†] and Eliot Fried[‡]

Cesare Davini Symposium
Udine, February 25-26, 2011

Abstract

When a thin rectangular sheet is clamped along two opposing edges and stretched, its inability to accommodate the Poisson contraction near the clamps may lead to the formation of wrinkles with crests and troughs parallel to the axis of stretch. A variational model for this phenomenon is proposed. The underlying energy functional includes bending and membranal contributions. Motivated by work of Cerda, Ravi-Chandar, and Mahadevan, the functional is minimized subject to a global constraint on the area of the midsurface of the sheet. Analysis of a boundary-value problem for the ensuing EulerLagrange equation shows that wrinkled solutions exist only above a threshold of the applied stretch. A sequence of critical values of the applied stretch, each element of which corresponds to a discrete number of wrinkles, is determined. Whenever the applied stretch is sufficiently large to induce more than one wrinkle, previously proposed scaling relations for the wrinkle wavelength and amplitude are confirmed. Comparisons with experimental measurements and numerical results indicate that the analytical results are remarkably robust.

[*]University of Udine, IT. E-mail: eric.puntel@uniud.it
[†]University of Trento, IT. E-mail: deseril@ing.unitn.it
[‡]McGill University, QC, Canada. E-mail: eliot.fried@mcgill.ca