Limit models for low, mean and high frequencies of a layered beam

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The objective of our present work is to obtain a mathematical model of the dynamical behavior of a three-layers beam and characterize its natural frequencies. We consider a two-dimensional beam consisting of three thin layers made of linear elastic isotropic materials: the upper and lower layers are called adherents, the middle layer is called adhesive. In order to perform an asymptotic analysis of the dynamical problem we choose a *small* real parameter ε which is used to scale respectively the thicknesses and the elastic moduli of the three layers. The thicknesses of the adherents are of order ε, while the thickness of the adhesive, being thinner than the upper and lower layers, is scaled with ε^2. The elastic moduli of the upper and lower layers remain unscaled, while the Lamé's constants of the middle layer are scaled with ε^2 since the adhesive is considered to be softer that the adherents. The asymptotic expansion method has been employed to formally justify classical theories of beams [1] or plates [2]. Different models of layered beams [3] have already been studied in the static case. The dynamical behavior of thin structures have been investigated by means of the asymptotic methods by [4] for beams and by for [2] for plates. The work of J.-L. Davet [4] deals with the derivation of limit models and natural frequencies for a single layer strip by using the asymptotic methods: the author characterizes the free vibrations of the beam for *low*, *mean* and *high* frequencies. In the present work the results of [4] are extended for a three-layers beam by rigorously characterizing the corresponding limit problems and limit natural frequencies.

References

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