Continuous Ladder Model of Fractional Viscoelasticity

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Viscoelastic materials have been more and more used nowadays for their low-cost productions as well as for their dissipative capabilities that may be coupled with others, more performing materials, to form complex-type engineering elements. The main feature of viscoelastic behavior is the relaxation of the stress state and the creep of the strain field that may be experienced, respectively, in hard or soft test devices. Such phenomenological consideration has been extensively analyzed yet at the beginning of the twentieth century and simple rheological models representing linear, constitutive, stress-velocity relations of the studied material have been proposed. Moreover the rheological relations have been also represented by a linear mechanical model, represented by a linear, viscous, dashpot relating the relative speed of its components to the applied load by a viscous coefficient. Combination of the dashpot, respectively in parallel or in series with a linear elastic spring corresponds to the Kelvin-Voigt or to the Maxwell model, respectively.

Large use of linear viscoelastic materials have been reported in scientific literature and engineering applications yet at the end of the fifties of the last century [1, 2]. These applications used the high damping characters of viscoelastic materials to provide passive controls of engineering systems in the form of bearing support or artificial dampers. The main feature of the viscoelastic material model provided with the linear models, either Kelvin-Voigt and Maxwell models, is related to the presence of a relaxation time that is characteristic parameter of the model.

Despite their wide diffusion, linear-type viscoelastic models do not match experimental evidences and suitable modifications of the rheological relations have been proposed in scientific literature as reported by several authors. Such improvements of the rheological relations have been obtained generalizing the Kelvin-Voigt model with different combinations of other mechanical elements, either dashpots or linear springs. In this way the mechanical response is obtained as combination of the responses of the various elements and the presence of multiple relaxation times is involved. The generalization of such an approach, with an infinite number of Kelvin-Voigt or Maxwell elements yields a response function obtained as a convolution integral of an exponential-type function, as representing the kernel, and the external load applied to the element.

A different choice of the kernel may, in the form of a power-law decay, corresponds to a Riemann-Liouville fractional derivative of the external load [3]. Such a model of fractional viscoelasticity has been introduced, recently, to represent the rheological behavior of more complexes viscoelastic media. The fractional model of viscoelasticity, introduced on mathematical basis, possesses an equivalent self-similar mathematical structure so that a fractal-type mechanical model has been introduced to represent such a kind of rheological model [4].

In this paper an innovative rheological model of viscoelastic material will be proposed based on the theory of long-range interactions introducing a point-spring-dashpot system that yields, as a continuous counterpart, a space fractional differential equation. In presence of a concentrated load the solution of the fractional differential equation yields a power-law time decay of the displacement of the mechanical system. Such a decay is corresponding, for a proper selection of model parameters to the Nutting power-law decay [5] showing that it corresponds to the constitutive relation expressed by time-varying fractional derivatives.

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