# Fiber distributed hyperelastic modeling of biological tissues

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## Sommario:

Several micro-structured biological tissues are characterized by anisotropy. The dependence of stiffness and strength on the orientation is often determined by the presence of cable-like micro- and nano-structures made of collagen. Recent findings concerning the arrangement of the structural collagen in biological tissues suggest that, although the functionality would require a prevailing orientation of the fibers, the organization of the organ introduces a certain degree of dispersion.
In many biological tissues, anisotropy manifests at a sustained level of strain. When they are straightened, collagenous cables are recruited to increase the material stiffness. Most anisotropic material models include the collagen contribution by defining a privileged direction that eventually modifies with the kinematics of the system.
Calculations available in the recent literature are more than satisfactory in qualitative sense, i.e. they provide realistic average values of stress fields. Quantitatively, though, there is not absolute certainty about the local value of the computed stresses. Too many factors affect the value of the stress in an organ, in addition to the individual specificity of the materials. Given the strong geometric and material nonlinearities, every material model provides different responses, and the same model may provide similar responses for different values of the parameters. Numerical calculations have demonstrated that the use of strongly oriented anisotropy models may lead to not realistic high concentration of stresses.
To achieve a better applicability to biological systems, anisotropic material models must reduce the level of determinism and account for the natural variability of tissues. A first step consists in considering statistical distributions of the fiber orientation and, as a consequence, of the mechanical properties. In this sense, exploiting the strength and simplicity of structure tensors, Gasser et al. [1] proposed a model based on a linear mixture between isotropic and strongly aligned two-fiber materials.
The generalized structure tensor model behaves quite well when two particular situations are considered. The first case is when the fibers are fully aligned; thus the model reduces to the strongly aligned fiber model [2]. The second case is when a uniform stretch is applied, so that the average stretch is equal to the uniform stretch. In many other cases, such as uniaxial, or shear, or biaxial loading, of dispersed fiber tissues, the error with respect exactly integrated distributed models may become relevant, up to 100%.
In view of a more realistic description of the spatial distribution of the collagen fibers in soft biological tissues, we propose an alternative material model [3], where the strain energy function depends on the mean value and on the variance of the pseudo-invariant $I_4$ of the distribution of the fibers. Indeed, the mean value was the only term considered in the generalized structure tensor model proposed in [1].
We derive the expression of the stress and of the consistent tangent stiffness of the new model and compare its mechanical response with the one of the original model for standard uniaxial, shear and biaxial tests. The comparisons are made with reference to the response of the exact fiber dispersed model, based on the direct integration of the contribution of the fibers.

### Bibiliografia

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