From 3D non-linear elasticity to 1D elastic models for thin walled beams

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Geometrically, a thin-walled beam is a slender structural element whose length is much larger than the diameter of the cross-section which, on its hand, is larger than the thickness of the thin wall. Beams of this kind have been used for a long time in civil and mechanical engineering and, most of all, in flight vehicle structures because of their high ratio between maximum strength and weight. Because of their slenderness thin-walled beams are quite easy to buckle and to deform and hence, in several circumstances, their study has to be conducted by means of nonlinear theories.

In this talk, starting from three-dimensional nonlinear elasticity, we rigorously derive a hierarchy of one-dimensional models for a thin-walled beam, in the spirit of what has been done by Friesecke, James and Müller in [2] for plates. The different limit models are distinguished by the different scaling of the elastic energy, which, in turn, depends on the scaling of the applied loads. More precisely, denoting by h and δ_h the length of the sides of the cross section and by ε_h the scaling factor of the bulk elastic energy, we can identify three main regimes:

- subcritical: $\delta_h/\varepsilon_h \stackrel{h\to 0}{\longrightarrow} 0$;
- critical: $\delta_h/\varepsilon_h \stackrel{h\to 0}{\longrightarrow} 1$;
- supercritical: $\delta_h/\varepsilon_h \stackrel{h\to 0}{\longrightarrow} +\infty$.

In the subcritical regime it is easy to see that, if $\varepsilon_h = O(1)$, then the limit model coincides with the nonlinear string model deduced by Acerbi, Buttazzo and Percivale in [1] for a beam of uniformly small cross-section. If instead $\lim_{h\to 0} \varepsilon_h = 0$, we obtain the energy of an inextensible string. In the critical regime, and under the additional assumption that $\lim_{h\to 0} h^2/\delta_h = 0$, we obtain a Cosserat model for a thin-walled beam. Finally, in the supercritical regime we deduce one dimensional linear/quasi-linear models for thin-walled beams.

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