A homogenized model for honeycomb cellular materials

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At mesoscopic scale, cellular materials are discrete materials characterized by a more or less clearly distinguishable architecture. In real life they form a large class and have a great variety of engineering applications. In most cases, their use serves macroscopic purposes, so that phenomenological properties are in fact of importance, and adopting a continuum description is obviously natural.

The construction of a continuum model is hindered by two types of difficulties: spatial variability of size and morphology of the microscopic architecture, on one side, and the crucial passage from the microscopic discrete description to the coarse continuum one, on the other. In this paper we focus on the passage from discrete to continuum and deduce the constitutive model for the in–plane deformations of a honeycomb material. Assuming periodicity of the microscopic array, we treat the problem within the framework of linear elasticity and assume that the underlying microstructure is governed by classical beam theory. The issue is relevant in view of the many applications of such materials and in particular for new materials, as *graphene*, where the assumption of geometric order is not a mere simplification.

Various authors have undertaken this problem, although under different assumptions, techniques and scope. Among them, Chen Huang and Ortiz [2], who derived a continuum model by means of an asymptotic Taylor's expansion of the displacement and rotation fields in the strain energy function of the system; Mourad, Caillerie and Raoult [1], who introduced a so called asymptotic homogenization technique and got a continuum expression of the energy integral by starting from discrete sums; and, finally, Dos Reis and Ganghoffer [3], who elaborated a variant of that technique, obtaining stress-strain relations under compression and shear for a 2D hexagonal lattice composed of extensional and flexural elements.

Here we adopt the viewpoint of homogenization theory and regard the continuum as the variational limit of a sequence of discrete systems consisting of an array of equal hexagonal cells of increasingly small size and wall thickness. The homogenized model emerges rather plainly from general theorems of Γ-convergence. At variance with the conclusions of [2], it turns out that the limit model is a *pseudo polar continuum*, that is, a material that can stand non traditional loading, such as applied distributed couples, but without developing couple stresses. It is likely that one can envisage couple stresses, at a finer level, by carrying out a functional asymptotic expansion of the type introduced by Anzellotti and Baldo, but this possibility is not investigated. We calculate the effective constitutive relations for general stress-strain states. The expressions found for the Young modulus and the Poisson ratio coincide with those calculated by Dos Reis and Ganghoffer and agree, in the limit, with formulae obtained by Gibson [4] and Gibson and Ashby [5]. In particular, the effective material is isotropic as in Gibson and Ashby's analysis. Yield and elastic buckling domains are also calculated. It is worth noticing that strength and stability thresholds do depend upon the stress state in an anisotropic way.

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