Time-asymptotic constitutive modeling of electrical conduction in biological tissues

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We study the electrical conduction in biological tissues in the radiofrequency range. In this context, a model was obtained by our group via homogenization theory [1]. It is governed by the following elliptic equation with memory for the electric potential $u_0(x,t)$:

$$-\operatorname{div}\left(A\nabla u_0 + \int_0^t B(\tau)\nabla u_0(x,t-\tau)\,\mathrm{d}\tau - \mathcal{F}\right) = 0\,, \quad \text{in } \Omega\times(0,+\infty), \tag{1}$$

where Ω is an open connected bounded subset of $\boldsymbol{R}^N$, A is a symmetric positive definite constant matrix, $B(t)$ is a symmetric matrix and $\mathcal{F}(x,t)$ is a vector. Moreover, $\|B(t)\|$ and $\|\mathcal{F}(\cdot,t)\|_{L^\infty(\Omega)}$ exponentially decay in time. Equation (1) is complemented with a Dirichlet boundary condition

$$u_0(x,t) = \Psi(x)\Phi(t)\,, \quad \text{on } \partial\Omega\times(0,+\infty), \tag{2}$$

where $\Phi(t)$ is a T-periodic function, for some fixed $T>0$, and $\Psi(x)$ is a smooth function.

The well-posedness of Problem (1)–(2) is proved in [2]. We are interested here in the behavior of the solution u_0 for large times. In this regard, we prove the following asymptotic stability result:

$$\|u_0(\cdot,t) - u_0^{\#}(\cdot,t)\|_{L^2(\Omega)} \le C\,\mathrm{e}^{-\lambda t} \quad \text{a.e. in } (1,+\infty), \tag{3}$$

where C and λ are positive constants and the function $u_0^{\#}(x,t)$ is the unique solution T-periodic in time of the equation:

$$-\operatorname{div}\left(A\nabla u_0^{\#} + \int_0^{+\infty} B(\tau)\nabla u_0^{\#}(x,t-\tau)\,\mathrm{d}\tau\right) = 0\,, \qquad \text{in } \Omega\times\boldsymbol{R}, \tag{4}$$

complemented with the boundary condition (2). Hence, roughly speaking, u_0 exponentially approaches a time-periodic steady state $u_0^{\#}$ as time increases, provided that a time-periodic Dirichlet boundary condition is assigned.

This work is relevant from the point of view of applications to electrical impedance tomography [3]. It could be also of interest in the mechanical modeling of composite materials with interfaces exhibiting a time-dependent behavior.

References

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